USE OF A DISLOCATION MODEL FOR DESCRIPTION OF SHOCK-LOADED RIGIDLY PLASTIC MEDIA WITH HARDENING

Yu. I. Meshcheryakov and V. A. Morozov

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In recent years, a considerable number of pieces of work on the study of the properties of a material as a function of the deformation rate have been devoted to the development of the microscopic approach to the description of the shock behavior of materials (see, for example, the review [1]). This approach is based on the use of the dynamic properties of the dislocations in writing the determining equations. The closed system of equations used for the description of elastoplastic waves in the case of monaxial loading has the form

$$\rho u_t + \sigma_x = 0; \quad (1)$$

$$u_x + \varepsilon_t = 0; \quad (2)$$

$$\sigma_t - \rho c^2 \varepsilon_t = -F, \quad (3)$$

where u is the rate of displacement of the particles of the material; ε is the total deformation in the direction of the propagation of the wave; σ is the stress; ρ is the density of the material; c is the speed of sound (adiabatic); and F is the relaxation function, whose form determines the dependence of the rate of plastic deformation of the material on the density of the mobile dislocations and their velocity. The subscripts t and x denote differentiation with respect to the time and the longitudinal coordinate, respectively. In the case of a polycrystalline material the relaxation function has the form [2]

$$F = \frac{8}{3}\mu b v_* \left\{N_0 + \frac{3}{8}\frac{\alpha}{\mu}\left[(\lambda + 2\mu)\varepsilon - \sigma\right]\right\} \exp\left\{-\frac{\tau_0 + \frac{3}{8}\frac{H}{\mu}[(\lambda + 2\mu)\varepsilon - \sigma]}{\frac{3}{4}\left[\sigma - \left(\lambda + \frac{2}{3}\mu\right)\varepsilon\right]}\right\}, \quad (4)$$

where N_0 is the initial density of the dislocations; H is the hardening constant; v_* is the velocity of the transverse sound waves; α is the multiplication factor of the dislocations; b is the Burgers vector; τ_0 is the characteristic stagnation stress; and λ and μ are Lamé coefficients.

Solution of the system (1)-(4) by the method of characteristics allows of a quantitative analysis of the effect of the parameters of the dislocation structure on the decay of the elastic precursor of the wave [2]. A number of communications [3-6] report a numerical solution of the system (1)-(4) by the method of finite differences using the Neumann–Richtmayer artificial viscosity. Under these circumstances, the qualitative and quantitative effect of the parameters of the dislocation structure on the whole profile of an elastoplastic wave was brought out.

A constructive method, making possible an unambiguous determination of the kinetic parameters of the dislocation structure – N_0, H, α, and τ_0 – on the basis of a quasi-steady-state dependence σ(ε), is developed in [6]. Values of the parameters obtained by successive approximations, by substitution into Eq. (1)-(3), gave rather close agreement between theoretical curves of the decay of an elastic precursor and the experimental results of [7], while the monotonic character of the dependence of the values of these parameters on the grain size is evidence of the correctness of the model.

In the present article, a somewhat different method is proposed for determining the kinetic parameters of the dislocation structure; it is based on an analytical solution of the system (1)-(3). As will be shown below, this solution yields an analytical connection between the steady-state front of an elastoplastic wave and the kinetic parameters for several types of relaxation functions F.

We first carry out the solution for the case of a polycrystalline material, taking the function of the relaxation in the form (4). As a new variable, we take the plastic shear deformation, which is connected with the normal stress and deformations by the relationship

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$$\gamma = -\frac{3}{8}\frac{1}{\mu}\left[\sigma - (\lambda + 2\mu)\,\varepsilon\right]. \tag{5}$$

Here the determining equation (3) can be converted to the form

$$\gamma_t = A(M\gamma + 1)\exp\left(-\frac{\tau + B\gamma}{\varepsilon - 2\gamma}\right), \tag{6}$$

where $A = bv_* N_0$; $B = H/\mu$; $M = \alpha/N_0$; and $\tau = \tau_0/\mu$.

From expression (6), the total (elastic plus plastic) deformation in the direction of propagation of the wave is equal to

$$\varepsilon = -\frac{\tau + B\gamma}{\ln\left[\gamma_t/A(M\gamma + 1)\right]} + 2\gamma. \tag{7}$$

Equations (1)-(2) can be reduced to one equation of the second order in σ and ε:

$$\sigma_{xx} - \rho\varepsilon_{tt} = 0, \tag{8}$$

which, taking account of (5), assumes the form

$$-(8/3)\mu\gamma_{xx} + \rho c^2\varepsilon_{xx} - \rho\varepsilon_{tt} = 0.$$

Substituting the deformation ε from expression (7) into this equation, we obtain an equation of the third order in partial derivatives with respect to the plastic shear deformation:

$$\begin{aligned}
&-\frac{8}{3}\mu\gamma_{xx} + \frac{\rho c^2(\tau + B\gamma)}{\left[\ln\dfrac{\gamma_t}{A(M\gamma+1)}\right]^3}\left\{\left[\frac{2B\gamma_x\gamma_{tx}}{(\tau + B\gamma)\gamma_t} - \frac{2BM\gamma_x^2}{(M\gamma+1)(\tau+B\gamma)} + \frac{\gamma_{txx}}{\gamma_t} -\right.\right.\\
&\left. - \frac{\gamma_{tx}^2}{\gamma_t^2} - \frac{M\gamma_{xx}}{M\gamma+1} + \frac{M^2\gamma_x^2}{(M\gamma+1)^2}\right]\ln\frac{\gamma_t}{A(M\gamma+1)} - \frac{B\gamma_{xx}}{(\tau+B\gamma)}\left[\ln\frac{\gamma_t}{A(M\gamma+1)}\right]^2 -\\
&\left. - \frac{2\gamma_{tx}^2}{\gamma_t^2} + 4\frac{M\gamma_{tx}\gamma_x}{\gamma_t(M\gamma+1)} - 2\frac{M\gamma_x^2}{(M\gamma+1)^2} + \frac{2\gamma_{xx}}{\tau+B\gamma}\left[\ln\frac{\gamma_t}{A(M\gamma+1)}\right]^3\right\} -\\
&- \frac{\rho(\tau + B\gamma)}{\left[\ln\dfrac{\gamma_t}{A(M\gamma+1)}\right]^3}\left\{\left[\frac{2B\gamma_{tt}}{\tau+B\gamma} - \frac{2BM\gamma_t^2}{(M\gamma+1)(\tau+B\gamma)} + \frac{\gamma_{ttt}}{\gamma_t} - \frac{\gamma_{tt}^2}{\gamma_t^2} + \frac{M^2\gamma_t^2}{(M\gamma+1)^2} -\right.\right.\\
&\left. - \frac{M\gamma_{tt}}{(M\gamma+1)}\right]\ln\frac{\gamma_t}{A(M\gamma+1)} - \frac{B\gamma_{tt}}{\tau+B\gamma}\left[\ln\frac{\gamma_t}{A(M\gamma+1)}\right]^2 - 2\frac{\gamma_{tt}^2}{\gamma_t^2} + 4\frac{M\gamma_{tt}}{M\gamma+1} -\\
&\left. - 2\frac{M^2\gamma_t^2}{(M\gamma+1)^2} - \frac{2\gamma_{tt}}{\tau+B\gamma}\left[\ln\frac{\gamma_t}{A(M\gamma+1)}\right]^3\right\} = 0,
\end{aligned} \tag{9}$$

where $\gamma_t = \partial\gamma/\partial t$; $\gamma_x = \partial\gamma/\partial x$; $\gamma_{tx} = \partial^2\gamma/\partial t\partial x$; ... This equation can be written in a more graphic form if the following notation is introduced:

$$\begin{aligned}
&\delta = \ln\left[\gamma_t/A(M\gamma + 1)\right], \quad \delta_t = \gamma_{tt}/\gamma_t - M\gamma_t/(M\gamma + 1), \ \delta_x = \gamma_{tx}/\gamma_t - M\gamma_x/(M\gamma + 1),\\
&\delta_{tt} = \gamma_{ttt}/\gamma_t - \gamma_{tt}^2/\gamma_t^2 - M\gamma_{tt}/(M\gamma + 1) + M^2\gamma_t^2/(M\gamma + 1)^2.
\end{aligned} \tag{10}$$

Using expressions (10), Eq. (9) is transformed in the following manner:

$$\begin{aligned}
&-\frac{8}{3}\mu\frac{\delta^3}{\rho c^2}\gamma_{xx} + 2B\delta\left(\gamma_x\delta_x - \frac{1}{c^2}\gamma_t\delta_t\right) + \delta(\tau + B\gamma)\left(\delta_{xx} - \frac{1}{c^2}\delta_{tt}\right) -\\
&- \delta^2 B\left(\gamma_{xx} - \frac{1}{c^2}\gamma_{tt}\right) - 2(\tau + B\gamma)\left(\delta_x^2 - \frac{1}{c^2}\delta_t^2\right) + 2\delta^3\left(\gamma_{xx} - \frac{1}{c^2}\gamma_{tt}\right) = 0.
\end{aligned} \tag{11}$$

We shall seek the solution of this equation in the form $\gamma = f(x - \alpha t)$, that is

$$M\gamma + 1 = M_0\exp(-kx + \omega t). \tag{12}$$

Then, as follows from expressions (10),

$$\delta_t = \delta_x = \delta_{tt} = \delta_{xx} = 0$$

and substitution of (12) into Eq. (11), taking account of (10), leads to the equation

$$\delta = \ln\frac{\omega}{AM} = B\Bigg/\left(2 - \frac{8}{3}\frac{\mu}{\rho}\frac{1}{c^2 - \dfrac{\omega^2}{k^2}}\right), \tag{13}$$

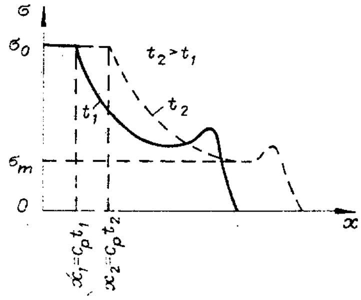

Fig. 1

TABLE 1

Material	Elastoplast. wave param.				Param. of struct. from plast. front			Param. of struct. from elast. precur.		Source
	c_p, mm/μsec	c, mm/μsec	σ_0, kbar	σ_m, kbar	N_0, cm^{-2}	α, cm^{-2}	H, kbar	N_0, cm^{-2}	τ_0, kbar	
Armco iron:										
Impact No. 5694	4,65	6,0	37,4	6,8	$1{,}02\cdot10^8$	$1{,}1\cdot10^{10}$	90	$2\cdot10^8$	19,8	[2]
Impact No. 5650	5,05	6,0	87,5	6,8	$0{,}7\cdot10^8$	$0{,}42\cdot10^{10}$	410	$2\cdot10^8$	19,8	[2]
NaCl	3,36	4,78	2,5	0,21	$1{,}9\cdot10^9$	$1{,}13\cdot10^{10}$	19,7	$1{,}5\cdot10^9$	0,422	[9]
LiF	4,95	6,63	29,3	3,6	$1{,}12\cdot10^9$	$1{,}28\cdot10^{10}$	100	$1{,}5\cdot10^9$	10	[10]

whence

$$k=\frac{\omega}{c}\Bigg/\left(1-\frac{8}{3}\frac{\mu}{\rho c^2}\frac{1}{2-\dfrac{B}{\ln\dfrac{\omega}{AM}}}\right)^{1/2}. \tag{14}$$

Here the shear deformation can be written in the form

$$\gamma=\frac{1}{M}\{M_0\exp[(AM\,e^{\delta}t)-kx]-1\}, \tag{15}$$

where δ and k are determined by expressions (13) and (14), respectively. The preexponential factor M_0 can be determined from the initial conditions $\gamma=0$ for $x=0$ and $t=0$, whence $M_0=1$.

The total deformation in the direction of propagation of the wave and the normal shear can be written in the form

$$\varepsilon=-\frac{\tau}{\delta}+\frac{2-B/\delta}{M}[\exp(\omega t-kx)-1]; \tag{16}$$

$$\sigma=-\frac{\tau}{\delta}\rho c^2+\frac{1}{M}\left[\rho c^2\left(2-\frac{B}{\delta}\right)-\frac{8}{3}\mu\right][\exp(\omega t-kx)-1]. \tag{17}$$

The sense of the expressions obtained can be brought out from Fig. 1, which shows the profile of an elastoplastic wave as a function of the longitudinal coordinate x. The stress at the plastic front of the wave starting from the coordinate $x=c_pt$ falls exponentially from a value of σ_0, equal to the initial stress applied at the boundary $x=0$ at the moment $t=0$, to some constant value σ_m, determined by the parameters of the material. The coordinate of the start of the fall of the plastic front is displaced in the positive direction of the x axis at the rate c_p. Thus, the value of c_p determines the velocity of the plastic front. The stress at the plastic front is equal to

$$\sigma=\sigma_0+(\sigma_0-\sigma_m)\{\exp[-k(x-c_pt)]-1\}\quad\text{for}\quad x\geqslant c_pt, \tag{18}$$
$$\sigma=\sigma_0\quad\text{for}\quad x<c_pt.$$

Conditions (18) determine the rate of fall of the stress at the plastic front:

$$\sigma_0=-\frac{\tau}{\delta}\rho c^2; \tag{19}$$

$$\omega=AM\exp\left(-\frac{\tau}{\sigma_0}\rho c^2\right). \tag{20}$$

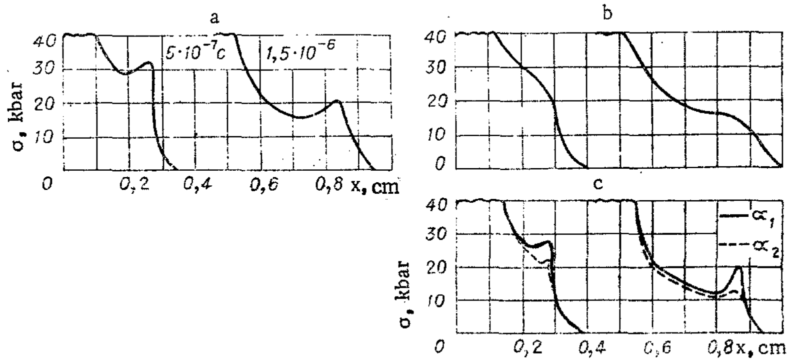

Fig. 2

For a known velocity of the plastic front c_p, expression (16), taking account of (19), determines the value of the coefficient of deformation hardening:

$$B = \frac{\tau}{\sigma_0}\rho c^2\left(\frac{8}{3}\frac{\mu}{\rho}\frac{1}{c^2 - c_p^2} - 2\right). \tag{21}$$

Finally, a comparison of expressions (18) and (19) yields a connection between the coefficient of multiplication of the dislocations and the stress σ_m at the minimum of the profile of the elastoplastic wave:

$$M = \frac{8}{3}\mu/(\sigma_0 - \sigma_m)\left(\frac{c^2}{c_p^2} - 1\right). \tag{22}$$

Thus, if the velocity of the plastic front c_p, the initial stress σ_0, the stress σ_m, the initial density of the dislocations N_0, and the stagnation constant τ_0 are known, then expressions (20)-(22) uniquely determine the remaining parameters of the dislocation structure of the material: the multiplication coefficient of the dislocations $\alpha = N_0 M$, the coefficient of hardening of the material $H = \mu B$, the increment of the growth of the plastic front of the wave

$$\omega = bv_* \alpha \exp\left(-\frac{\tau_0}{\mu\sigma_0}\rho c^2\right), \tag{23}$$

and the wave number

$$k = \omega / c_p.$$

The values of the characteristic stagnation stress τ_0 and the initial density of the mobile dislocations can be, as is well known [2], determined from data on the decay of the elastic precursor, using the method of characteristics. The parameters σ_0, σ_m, and c_p can be determined from experiments on shock loading. Thus, using expressions (19)-(23), we can determine the averaged characteristics of the rate of multiplication of the dislocations and the hardening coefficient of the material.

However, as is noted in a number of communications [8-10], the value of the initial density of the dislocations N_0, flowing out of data on the decay of the elastic precursor, is found to be 2-3 orders of magnitude greater than that observed from etching pits. In view of this, it is reasonable to determine N_0 from an analytical solution of (18), selecting the parameters τ_0, N_0, α, and H in such a way that the calculated plastic front of the wave will coincide with the experimental. An analysis of the wave profiles in the experiments of [7] gives the values of the parameters shown in Table 1. Here the value of the characteristic stagnation stress τ_0 was taken equal to 19.8 kbar, and the dynamic equilibrium amplitude of the shock wave is determined from the asymptotic velocity of the free surface $u_{fs} = 0.028$ mm/μsec, using the Hugoniot–Rankine relationship $\sigma_m = (1/2)\rho c u_{fs}$. As can be seen from the data given in the table, the initial density of the dislocations, determined from the condition of the coincidence of the plastic fronts, corresponds to values obtained from analysis of the data of [2] on the decay of the elastic precursor.

To clarify which dependence between σ and ε corresponds to the analytical solution (17) obtained above, we return again to the determining equation (6). As has been noted above, a solution of the system (1)-(4) is possible only for $\delta = \text{const}$, where it becomes linear. A comparison of expressions (6) and (10) gives $\delta = -(\tau + B\gamma)/(\varepsilon - 2\gamma)$. Substituting σ into this expression by the replacement of γ in accordance with formula (5), we obtain the sought relationship in the form

$$\sigma = \rho c_p^2 \varepsilon + \frac{\tau}{B}\left[\frac{8}{3}\mu - 2\rho\left(c^2 - c_p^2\right)\right]. \tag{24}$$

Denoting the hydrostatic velocity in the medium by $c_h = [(c^2 - \frac{4}{3}(\mu/\rho))]^{1/2}$, expression (24) can be rewritten in the form

$$\sigma = \rho c_p^2 \varepsilon + \frac{2\tau\rho}{B}(c_p^2 - c_h^2). \tag{25}$$

Expressions (24) and (25) give a linear connection between σ and ε. The slope of the plastic part of this dependence is determined by the rate of propagation of the plastic front c_p, which is present as a parameter in expressions (24) and (25). Since the velocity of the plastic front is less than the acoustical velocity, the slope of the plastic part of the dependence $\sigma-\varepsilon$ is less than the slope of its elastic part, determined, as is well known, by the relationship $\sigma = \rho c^2 \varepsilon$. The least slope of the plastic part of the dependence $\sigma-\varepsilon$ occurs with equality of the plastic front and the hydrostatic velocity in the medium under consideration. Thus, the analytical solution of the system (1)-(4) obtained corresponds to the well-known rigidly plastic scheme, whose parameters are determined by the parameters of the dislocation structure of the material, in accordance with expression (24).

By an appropriate selection of (12), out of the whole class of solutions of the system (11), a solution is obtained satisfying the linear dependence $\sigma-\varepsilon$ in the plastic region of the dynamic stress diagram. A rigidly plastic scheme with hardening cannot take account of the dispersion of the wave parameters determining the profile of the plastic front. This means that the analytical solution obtained describes only fully established plastic fronts, the stress at any given point of which remains unchanged during the process of their propagation, while the fronts themselves, as a whole, are displaced with a velocity c_p, less than the velocity of the elastic precursor. The possibility of the existence of fully established plastic fronts has been repeatedly pointed out in the literature. Specifically, in [11], the conditions for the appearance of such fronts in aluminum are determined theoretically and experimentally.

The method for the analytical description of plastic fronts, developed in the present article, is essentially based on the replacement of a nonlinear dependence $\sigma-\varepsilon$ by a linear rigidly plastic dependence with hardening, whose inclination to the axis of abscissas is determined by the velocity of the plastic front. The value of the initial density of the dislocations, calculated from an analysis of the experiments of [7], is very close to the analytical value obtained on the basis of data on the decay of the elastic precursor. As can be seen from the calculated data given, from the profile of the plastic front, additional information can be obtained on the hardening of the material and the multiplication properties of the dislocation structure. The method has considerable advantages also in the sense that all the needed information on the averaged parameters of the dislocation structure can be extracted from one experimentally recorded profile of an elastoplastic wave, while, to plot the curve of the decay of the elastic precursor, a series of impacts is required for different thicknesses of the target.

Simultaneously, the system (1)-(4) was solved numerically, with different values of the parameters of the dislocation structure entering into the equations. The solution was obtained by the method of finite differences, with the introduction of the Neumann–Richtmayer artificial viscosity. Figure 2a-c shows profiles of the stress in the wave, calculated for two fixed moments: $t_1 = 5 \cdot 10^{-7}$ sec and $t_2 = 1.5 \cdot 10^{-6}$ sec; $\tau_0 = 19.6$ kbar, H = 4.1 kbar [a) $N_0 = 10^6$ cm^{-2}, $\alpha = 7 \cdot 10^{10}$ cm^{-2}; b) $N_0 = 10^8$ cm^{-2}, $\alpha = 7 \cdot 10^{10}$ cm^{-2}; c) $N_0 = 10^6$ cm^{-2}, $\alpha_1 = 3 \cdot 10^{10}$ cm^{-2}, $\alpha_2 = 3 \cdot 10^{12}$ cm^{-2}]. It can be seen that the most significant effect on the form of the elastoplastic wave is that of the initial density of the mobile dislocations. For $N_0 = 10^6$ cm^{-2}, a considerable decay of the elastic precursor, corresponding to the experiments of [7], can be attained only with the introduction of an anomalously large multiplication coefficient of the dislocations $\alpha = 3 \cdot 10^{12}$ cm^{-2}. The rather high values of the multiplication coefficient of the dislocations, obtained from numerical calculations, along with the extremely high density of the dislocations obtained independently from an analysis of the decay of the elastic precursor [2] and the analytical approximation of the data of the article, do not contradict the models developed in recent years for the heterogeneous multiplication of dislocations in the elastic precursor of a wave. In accordance with this model, a reasonable agreement of the rate of decay of the precursor with experiment is obtained with the introduction into the model of additional mechanisms (of an explosive character) of the generation of dislocations, responsible for the rapid relaxation of the stress and for the fall in the elastic precursor to the dynamically equilibrium value. The mechanism of this multiplication still remains unclear, although there are a number of data on the effect of the impurity composition of the material on it [10, 12].

The above solution of the system (1)-(3) describes the shock-wave behavior of polycrystalline materials. Experimental data obtained with the shock-wave loading of single crystals are more convenient for an analysis of the dislocation structure. In the case of single crystals, the form of the relaxation function is determined by the orientation of the crystal with respect to the direction of propagation of the wave. For different types of crystalline systems and directions of the impact, forms of the relaxation functions were obtained in [8]. Taking account of the data of this paper, in the general case the determining equation (3) can be written in the form

$$\sigma_t - \rho c^2 \varepsilon_t = -Rbv_* \{N_0 + \alpha[\rho c^2 \varepsilon - \sigma]\} \exp\{-[\tau_0 + H(\rho c^2 \varepsilon - \sigma)]/q\sigma\},$$

where the coefficients R and q are determined by the type of crystal and the direction of propagation of the wave (see [8]). For shear deformation, this equation assumes the form

$$\gamma_t = A(M\gamma + 1) \exp[-(\tau_0 + H\gamma)/q\sigma],$$

whence

$$\sigma = -(\tau_0 + H\gamma)/q \ln[\gamma_t / A(M\gamma + 1)]. \tag{26}$$

Expressing the total deformation in terms of the stress and the shear deformation, in accordance with formula (5), Eq. (8) can be converted to the form

$$\sigma_{xx} - (1/c^2)(\sigma_{tt} - R\gamma_{tt}) = 0.$$

Finally, substituting here the value of σ from expression (26), and again postulating a solution in the form (12), taking account of (10) we obtain the following dispersion equation:

$$k = \frac{\omega}{c} \sqrt{1 - \frac{Rq}{H} \ln \frac{\omega}{AM}},$$

in which the plastic shear deformation and the stress in the wave are determined by the expressions

$$\gamma = (1/M)\{\exp[k(c_p t - x)] - 1\},$$

$$\sigma = -\tau_0/q\delta - (H/q\delta M)\{\exp[k(c_p t - x)] - 1\}.$$

From the moment of time $t_1 = x/c_p$ (see Fig. 1), the stress remains constant and equal to σ_0 for all values of the coordinate $0 < x \leq x_1$. Thus, from the condition $\sigma = \sigma_0$ at $t = t_1$ we have

$$\sigma_0 = -\tau_0/q\delta, \quad \omega = AM \exp(-\tau_0/q\sigma_0).$$

From this expression, specifically, it can be seen that the rate of growth of the plastic front does not depend on the type of crystal lattice, but is determined only by the parameters of the dislocation structure of the material. For a known velocity of the plastic front, the hardening coefficient of the material is determined as

$$H = \frac{\tau_0}{\sigma_0} \frac{R}{c^2/c_p^2 - 1}.$$

The coefficient of multiplication of the dislocations $\alpha = N_0 M$ can be determined from the following condition (as $x \to \infty$, $\sigma \to \sigma_m$), determining the stress behind the elastic precursor:

$$M = R \bigg/ \left(\frac{c^2}{c_p^2} - 1\right)(\sigma_0 - \sigma_m).$$

The expressions obtained were used for an analysis of experimental data on the shock-wave loading of single crystals of table salt [9] and lithium fluoride [10]. The results of a calculation for a direction of the impact [100] are given in Table 1.

Thus, both for polycrystalline materials and for single crystals, the expressions obtained allow of an unambiguous determination of the averaged parameters of the dislocation structure, within the framework of the Gilman–Johnson model.

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LITERATURE CITED

1. J. J. Gilman, "Dislocation dynamics and the response of materials to impact," Appl. Mech. Rev., 21, No. 8, 767-783 (1968).
2. J. W. Taylor, "Dislocation dynamics and dynamic yielding," J. Appl. Phys., 36, No. 10, 3146-3150 (1965).
3. J. N. Johnson and W. Band, "Investigation of precursor decay in iron by the artificial viscosity method," J. Appl. Phys., 38, No. 4, 1578-1585 (1967).

4. M. L. Wilkins, "Calculation of elastoplastic flows," in: Computational Methods in Hydrodynamics [Russian translation], Mir, Moscow (1967).
5. A. I. Gulidov, V. M. Fomin, and N. N. Yanenko, "The structure of compression waves in inelastic media," Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela, No. 5, 92-98 (1975).
6. R. I. Nigmatulin and N. N. Kholin, "A model of an elastoplastic medium with dislocation kinetics of the plastic deformation," Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela, No. 4 (1974).
7. J. W. Taylor and M. H. Rice, "Elastic-plastic properties of iron," J. Appl. Phys., 34, No. 2, 364-371 (1963).
8. J. N. Johnson, O. E. Jones, and T. E. Michaels, "Dislocation dynamics and single-crystal constitutive relation shock-wave propagation and precursor decay," J. Appl. Phys., 41, No. 6, 2330-2339 (1970).
9. W. J. Murri and G. D. Anderson, "Hugoniot elastic limit of single-crystal sodium chloride," J. Appl. Phys., 41, No. 8, 3521-3525 (1970).
10. I. R. Asay, G. R. Fowles, G. E. Duvall, M. H. Miles, and R. F. Tinder, "Effect of point defect on elastic precursor decay in LiF," J. Appl. Phys., 43, No. 5, 2132-2145 (1972).
11. J. N. Johnson and L. M. Barker, "Dislocation dynamics and steady plastic wave profiles in 6061-T6 aluminum," J. Appl. Phys., 40, No. 11, 4321-4334 (1969).
12. Y. M. Gupta, G. E. Duvall, and G. R. Fowles, "Dislocation mechanisms for stress relaxation in shocked LiF," J. Appl. Phys., 46, No. 2, 532-540 (1975).

EQUATIONS OF ELASTOPLASTIC DEFORMATION FOR ARBITRARY VALUES OF THE ROTATIONS AND DEFORMATIONS

G. V. Ivanov

UDC 539.3

In many solids, for example, in metallic bodies, for arbitrary values of the rotations and deformations of the elements of the body, the components of the deviator of the elastic deformations are quantities on the order of the ratio of the shear strength to the Young modulus and, consequently, are small in comparison with unity. Below, on the basis of the results of [1], equations are formulated for the isotropic elastic and ideal elastoplastic deformation of such bodies. A comparison is made between the equations obtained and known equations [2-4]. For simplicity in writing the equations, only adiabatic deformations are discussed below.

1. Equations of Elastic Deformation in the Case of Small Components of the Deviator of the Deformations

We denote by $\mathbf{\partial}_\alpha$, $\mathbf{\partial}^\beta$ $(\alpha, \beta = 1, 2, 3)$ the basis vectors of a Lagrangian system of coordinates, generated by the Cartesian system of coordinates x^i with the basis vectors $\mathbf{k}_i = \mathbf{k}^i$ $(i = 1, 2, 3)$.

Let $\hat{\gamma}_{\alpha\beta}\mathbf{\partial}^\alpha\mathbf{\partial}^\beta = \hat{\gamma}^{\sigma\lambda}\mathbf{\partial}_\sigma\mathbf{\partial}_\lambda = \gamma_{ij}\mathbf{k}^i\mathbf{k}^j$ be some symmetrical tensor. Differentiating the formulas for the connection between the components $\hat{\gamma}_{\alpha\beta}$, $\hat{\gamma}^{\sigma\lambda}$ and the components γ_{ij}, we find

$$(d\hat{\gamma}_{\alpha\beta}/dt)\mathbf{\partial}^\alpha\mathbf{\partial}^\beta = (D\gamma_{ij}/Dt + \gamma_{sj}e_{si} + \gamma_{si}e_{sj})\mathbf{k}^i\mathbf{k}^j,$$
$$(d\hat{\gamma}^{\alpha\beta}/dt)\mathbf{\partial}_\alpha\mathbf{\partial}_\beta = (D\gamma_{ij}/Dt - \gamma_{sj}e_{si} - \gamma_{si}e_{sj})\mathbf{k}^i\mathbf{k}^j, \tag{1.1}$$

where $e_{ij} = (1/2)(\partial u_i/\partial x^j + \partial u_j/\partial x^i)$; u_i are the components of the velocity vector; $D\gamma_{ij}/Dt$ is a Jaumann derivative [5]

$$D\gamma_{ij}/Dt = d\gamma_{ij}/dt + \gamma_{ki}\omega_{kj} + \gamma_{kj}\omega_{ki},$$
$$\omega_{ij} = (1/2)(\partial u_i/\partial x^j - \partial u_j/\partial x^i).$$

From (1.1), specifically, it follows that

$$D\varepsilon_{ij}/Dt + \varepsilon_{ki}e_{kj} + \varepsilon_{jk}e_{ki} = e_{ij}, \tag{1.2}$$

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